Stress-coupling phenomena in anisotropic fibres

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An analytical framework is presented to interpret coupling between axial normal stresses and torsional shearing stresses observed in anisotropic fibres. This coupling can be uniquely described in terms of the material properties describing the anisotropy and the testing geometry. The coupling is described through a consideration of the rotation of material symmetry directions caused by shear and the application of standard transformation laws governing anisotropic elastic properties. The analysis is employed to describe the apparent linear increase in torsional modulus with axial tensile stress observed in anisotropic fibres. Isotropic fibres are predicted and observed to be free of such coupling. The results support the recent modelling of oriented polymer fibre compressive strength as an elastic buckling instability.

(Keywords: mechanical anisotropy; shear modulus; fibre torsion; compressive strength)

INTRODUCTION

While the tensile properties of various polymeric high-performance fibres are of primary interest, the anisotropy of mechanical properties that many of these fibres exhibit is also of relevance to many engineering applications as well as to the thorough characterization of the materials. Yet, few studies have been concerned with the determination of mechanical anisotropy in polymeric high-performance fibres. Some recent studies have begun to address this area by analysis[1,2] and by experiment via investigation of fibre torsional behaviour[1-6]. Torsional experiments have illustrated the high degree of anisotropy present in many of these polymeric high-modulus/high-strength fibres.

In order to characterize fully the mechanical anisotropy of a material, multiple tests are required involving simple stress states as well as more complicated stress states and combined states of stress. The characterization of anisotropy in high-performance fibres is experimentally complicated by restrictions imposed by the fibre geometry. The use of combined states of stress testing for fibres, however, would provide valuable mechanical property information as well as provide tests of various structural models that have been developed to describe the mechanical properties of the fibres.

The behaviour of various high-modulus/high-strength fibres was recently investigated in combined torsion and tension experiments as part of a dissertation by DeTeresa[5]. Coupling effects were observed in these experiments for anisotropic fibres whereas no coupling was found for an isotropic fibre. The coupling between tensions and torsions was interpreted empirically via a resistance to volume change. In the present work, a theoretical framework is developed to interpret these coupling effects in terms of the material constants (compliances, stiffnesses) describing the anisotropy of the fibre. Additional experimental results are also reported.

COUPLING ANALYSIS

In the most general case of anisotropy, coupling effects between axial normal strains and torsional shearing strains are expected[7,8]. If material symmetry arguments are imposed such that the fibre is no more complicated than cylindrically orthotropic (having three mutually perpendicular planes of symmetry, e.g. r, θ, z), then such coupling is not expected[7]. This is also the case for transversely isotropic and isotropic fibres. Coupling effects are expected and observed, however, even in orthotropic bodies when various off-axis tests are employed and care is not taken to ensure that a uniform state of stress is achieved[9].

In order to develop a framework for describing coupling effects in fibre testing, the case of torsional testing of a cylindrically orthotropic fibre under the combined influence of an axial load will be considered. To achieve these combined testing modes on fine-diameter fibres, the fibres are rigidly bonded or clamped at each end of a gauge length, at which points axial and torsional deformations are imposed. As simple shear is imposed upon the fibre, the axial plane of elastic symmetry is considered to be rotated away from the fibre axial direction by an amount equal to the torsional shear angle. Thus, the axial fibre loading can be considered to be an off-axis loading with respect to the rotated material symmetry directions. Potential coupling between torsion and axial stress may then be evaluated following the standard rules of transformation of the orthotropic elastic constants and the generalized Hooke's law.

Following standard engineering notation[7,8], Hooke's law may be expressed in terms of material compliances as:

$$\varepsilon_i = a_{ij}\sigma_j \qquad (1)$$

where σ_j are the stresses, ε_i the strains and a_{ij} the elastic compliances with a summation over index j implied in the

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usual way. The component of equation (1) describing the fibre torsional response for a cylindrically orthotropic fibre is:

$$\gamma = a_{66}\tau \tag{2}$$

where γ and τ are the shearing strain and stress, respectively. If now the rotation of the material symmetry directions is considered, equation (1) is to be replaced by:

$$\varepsilon_i = a'_{ij}\sigma_j \tag{3}$$

a'_{ij} represents the transformed elastic compliances accounting for the rotation. Equation (2) then becomes:

$$\gamma = a'_{16}\sigma_z + a'_{26}\sigma_r + a'_{36}\sigma_\theta + a'_{66}\tau \tag{4}$$

where the coupling terms a'_{16}, a'_{26} and a'_{36} are no longer absent. Neglecting the small radial and tangential stresses that may arise due to the cylindrical orthotropy[1], this further reduces to:

$$\gamma = a'_{16}\sigma_z + a'_{66}\tau \tag{5}$$

where a'_{16} and a'_{66} are defined from the transformation rules as[8]:

$$a'_{16} = [2a_{22}\sin^2\theta - 2a_{11}\cos^2\theta + (2a_{12} + a_{66})(\cos^2\theta - \sin^2\theta)]\sin\theta\cos\theta \tag{6}$$

$$a'_{66} = 4(a_{11} + a_{22} - 2a_{12} - a_{66})\sin^2\theta\cos^2\theta + a_{66}$$

with θ being the angle describing the rotation of the symmetry axis with respect to the loading direction and a_{ij} the elastic compliances describing the cylindrical orthotropy with respect to material symmetry directions.

Owing to the odd powers of the sine function involved in the coupling term a'_{16}, care must be taken in accounting for the proper sign of the transformation angle θ[8,10]. The angle θ is measured from the test direction z to the material symmetry axis. The magnitude of θ will be taken as the magnitude of the shear strain γ. That is, the material symmetry axis originally along z is assumed to be effectively rotated away from the axial direction as a result of shearing (*Figure 1*). For positive shear stress and strain, as in *Figure 1b*, θ is thus measured in a clockwise direction and is hence negative. For negative shear, as in *Figure 1c*, θ is measured in an anticlockwise or positive direction. Thus, θ is always of opposite sign to γ:

$$\theta = -\gamma \tag{7}$$

Considering only small shearing angles, we additionally make use of the following linearizations:

$$\cos\theta \sim 1$$

$$\sin\theta \sim \theta$$

Equations (5), (6) and (7) may then be combined to yield:

$$\gamma = a_{66}\tau - \gamma(2a_{12} - 2a_{11} + a_{66})\sigma_z \tag{8}$$

Rearranging and defining τ/γ as the apparent or measured shear modulus (G_m) yields:

$$G_m = \frac{1}{a_{66}} + \frac{1}{a_{66}}(2a_{12} - 2a_{11} + a_{66})\sigma_z$$

or in terms of the more common engineering constants:

$$G_m = G + \left(1 - \frac{2G(1+\nu)}{E}\right)\sigma_z \tag{9}$$

where E is the axial Young's modulus ($1/a_{11}$), G is the true shear or torsional modulus ($1/a_{66}$), and ν is Poisson's ratio.

The coupling between axial normal stress and torsional response described by equation (9) predicts an apparent linear increase in measured torsional modulus with applied axial stress. The material constant term enclosed in large parentheses in equation (9) provides a measure of the anisotropy of the fibre. For convenience, we may write equation (9) as:

$$G_m = G + A\sigma_z \tag{10}$$

where $A = [1 - 2G(1+\nu)/E]$ represents a measure of the fibre's anisotropy. For an isotropic fibre, A is identically zero by definition, whereas for highly anisotropic materials where G/E is typically a small number[1-3], A approaches a value of unity. This is consistent with the observations of DeTeresa[5] on isotropic and anisotropic fibres and provides an explanation of the effect based on the application of anisotropic linear elasticity rather than as an empirical resistance to volume change.

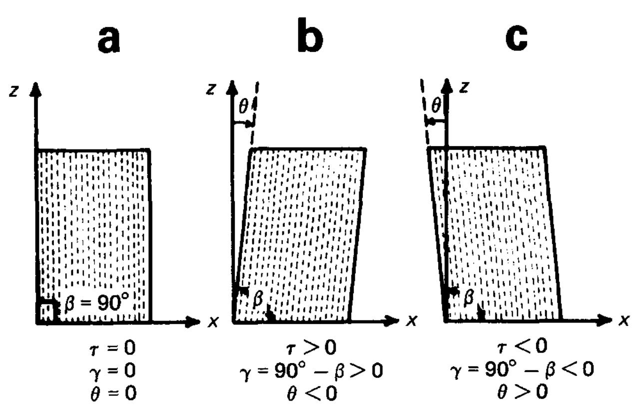

Figure 1 Geometry of shear deformation: (a) reference, (b) positive shear and (c) negative shear

EXPERIMENTAL

Fibre shear moduli were measured via free torsional oscillations using a variation of the equipment described in refs. 5 and 11. The apparatus is depicted in *Figure 2*. The fibre sample is first bonded using jewellers' wax onto a cardboard tab defining a total sample length of 100 mm. A torsion bar made from 20 gauge phosphor-bronze wire is then similarly bonded to the centre of the sample length. One end of the bonded sample is clamped in the top clamp of the apparatus and then the sides of the tab are carefully cut away to allow the sample to hang freely. Care is taken to allow the sample to come to rest to remove any twist that may have been imparted during mounting.

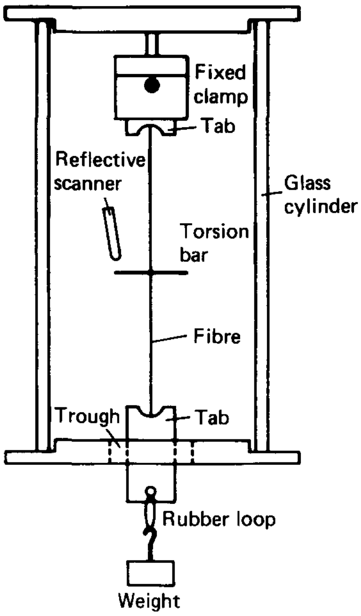

Figure 2 Torsion pendulum apparatus for combined loading

The sample is then lowered into the glass cylinder and the bottom tab guided through a rectangular slot in the lower cylinder end plate. The slot provides freedom of axial movement but not of rotational movement. Special care is taken to ensure that the sample hangs vertically and the bottom tab is carefully centred in the trough. Tension is applied to the fibre by suspending weights from the lower tab. Inertial effects due to vibration are minimized by employing a small rubber loop from which the tension weights hang.

The moment of inertia of the bar used was 2.765 g mm^2 and provided for periods of oscillation in the range of 5–20 s for the fibres tested. The torsion bar is set into motion by means of a guide bar through which a rotation of 90–180° is imparted. The period of oscillation is measured by a timer connected to a reflective scanner which detects the passage of the torsion bar past the zero position. Oscillations of the order of $\pm 90°$ were employed for the measurements. Four to ten periods were measured and averaged and then two to four repeat measurements made. The tension weight is changed and the measurements repeated.

After testing, the fibre sample was cut from the torsion bar and the tabs and both halves were then deniered using a Vibromat M[12]. An average fibre radius was then calculated from the denier and known density of the fibre. Single filaments used for these torsional tests were obtained by extracting filaments from the following yarns: type 30 glass roving (Owens Corning), type 717 nylon (Du Pont) and 'Kevlar'* 49 aramid (Du Pont).

RESULTS AND DISCUSSION

For a centrally bonded torsion bar, as indicated in *Figure 2*, the shear modulus for a fibre of circular cross-section is given by:

$$G_m = \frac{2\pi I_b L}{P^2 R^4} \tag{11}$$

* Du Pont registered trademark

where I_b is the moment of inertia of the torsion bar, L the total fibre length, R the fibre radius and P the period of oscillation. Any influence of axial tension on the torsional behaviour of the fibre would thus be manifested in a change in the period of torsional oscillation, all other factors in equation (9) being constant for a given sample. The experimental results are summarized in *Figures 3–5*.

Within experimental precision, the period of torsional oscillation of the glass filaments was found to be constant at all tensile loadings, including loads near the break strength (*Figure 3*). Thus the glass fibre shear modulus was unaffected by the simultaneous application of a tensile load. According to equation (10) the anisotropy term A describing coupling effects is equal to zero, characteristic of an isotropic fibre. This is in agreement with expectations for glass and with the observations of DeTeresa[5].

The periods of torsional oscillation for both the nylon and Kevlar® 49 filaments were found to decrease with increasing tensile load. *Figures 4* and *5* summarize the nylon and Kevlar® 49 filament behaviour, respectively. This decrease in period of oscillation is equivalent to an increase in the apparent or measured shear modulus as depicted in *Figure 6* for the Kevlar® 49 fibres. This observed change in shear modulus with axial load is consistent with the fibres being anisotropic, as described

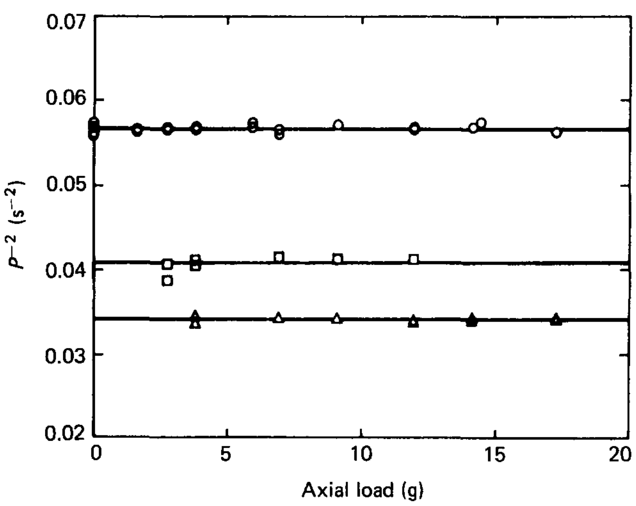

Figure 3 Torsional response of glass fibres: (○) 4.0 denier filament; (□) 3.47 denier filament; (△) 3.05 denier filament

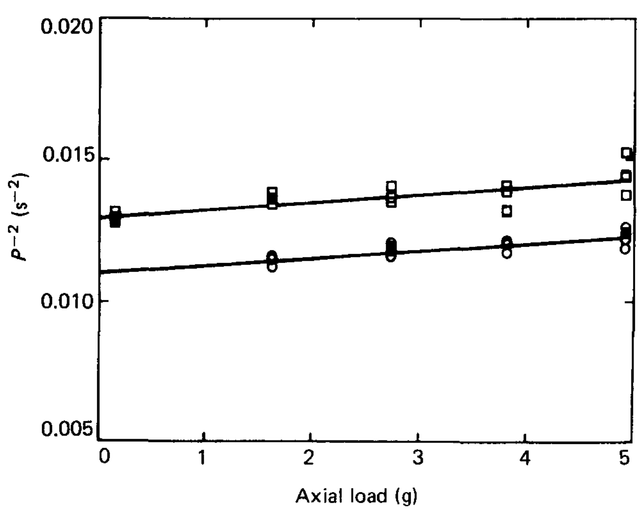

Figure 4 Torsional response of nylon fibres: (□) 6.1 denier filament; (○) 5.64 denier filament

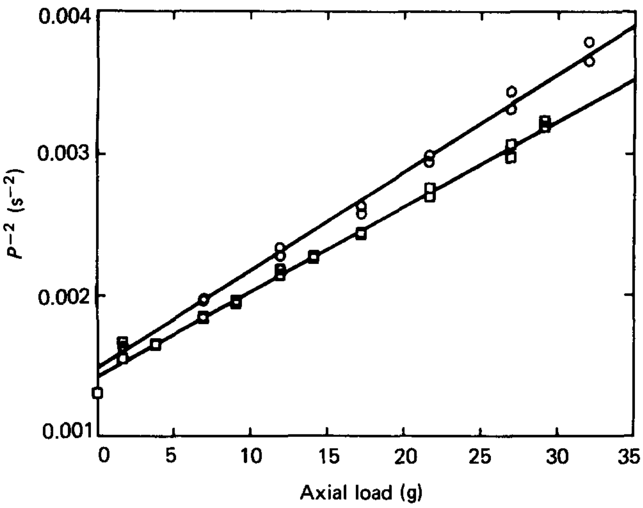

Figure 5 Torsional response of Kevlar® 49 filaments: (○) 1.56 denier filament; (□) 1.42 denier filament

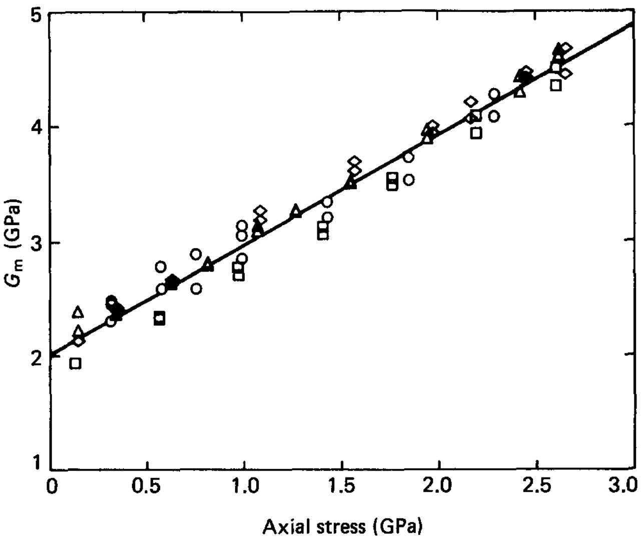

Figure 6 Calculated shear modulus dependence on axial stress for Kevlar® 49 fibres. The full line is a fit to equation (10). Symbols represent different individual filaments

by equation (10). That is, the anisotropy term A in equation (10) is non-zero. Linear regression analysis of the data permits a calculation of the anisotropy A as expressed in (10). The calculated results are given in *Table 1*, where the fibres are ranked from isotropic (glass) to highly anisotropic (Kevlar®).

The calculated values of anisotropy presented in *Table 1* have an accuracy no better than 20%, which prohibits a precise evaluation of Poisson's ratio for these fibres. The fourth-power dependence of the shear modulus on the fibre radius in these tests is the major source of uncertainty. These experiments do, however, provide an independent measure of the anisotropy term A, which also appears in recent models describing the tensile behaviour of oriented polymer fibres[13-16]. The value obtained for Kevlar® 49 is in good agreement with values used by Northolt[13-15] in modelling the deformation of poly(p-phenylene terephthalamide) fibres.

It is interesting to note one peculiarity that equations (9) and (10) suggest. If axial compression rather than axial tension were applied in these torsion experiments, then a decrease in the apparent shear modulus is expected. At a particular value of compressive stress, the apparent shear modulus would become zero. In the case of a highly anisotropic fibre, where the anisotropy term approaches a value of unity, this compressive stress would be equal in magnitude to the shear modulus G. At this stress level, an instability is expected as G_m becomes zero. This effect supports recent theoretical work[5,17] that describes the compressive strength of oriented polymers as a buckling instability predicted to occur at a compressive stress of magnitude equal to the shear modulus.

The theoretical framework presented provides a means of evaluating fibre anisotropy from combined stress testing. This framework may be easily extended to other types of loadings to describe behaviour based on material properties and testing geometries. Further evaluation of anisotropic fibre behaviour under multiple stress states in light of similar analysis may aid critical evaluation of structural models proposed to describe fibre mechanical properties.

Table 1 Summary of fibre torsional behaviour

Fibre	Density ($g\,cm^{-3}$)	E (g/denier)	E (GPa)	G (g/denier)	G (GPa)	A
Glass	2.5	300	65	135	30	0
Nylon	1.14	55	5.5	6.2	0.62	0.78
Kevlar® 49	1.44	930	120	16	2	0.95

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